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Surface pion condensation in finite nuclei

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Abstract. We discuss here the possible occurrence of surface pion condensation in finite nuclei. We show the argument for this possibility by the recent findings in few-body calculations and the spin-flip experiments. We show the calculated results on various $N = Z$ nuclei using the relativistic mean-field theory with a pion.

PACS. 21.60.-n Nuclear-structure models and methods

1 Introduction

The pion was conjectured by Yukawa as the mediator of the nucleon-nucleon interaction [1]. It has the properties $J^\pi = 0^-$ spin parity and $T = 1$ isospin, which result in rich phenomena in hadron and nuclear physics. The pion plays a central role in hadron physics, particularly for low-energy phenomena. The chiral perturbation approach, with the pion acting as the essential degree of freedom, is a powerful tool in the study of hadron properties and their interactions.

Since the proposal of the shell model, nuclear structure has been studied in terms of the single-particle basis [2]. In that treatment, single-particle orbits are obtained with the parity-conserved mean field given by the central and spin-orbit potential. However, when a virtual pion is emitted and absorbed by a nucleon in a good-parity orbit, the nucleon makes a jump from one single-particle state to another with opposite parity, accompanied by a spin flip due to the pion-nucleon coupling. Therefore, to incorporate the effect of the correlation caused by the pion exchange interaction in the parity-conserved single-particle space, we must treat higher configurations, like 2p-2h (2 particle-2 hole) states, crossing over major shells. To avoid such complications, we renormalize the central and the spin-orbit interactions in nuclei to take into account the strong correlations caused by the pion in the restricted model space. This means that we assume a model space in which the parity of a single-particle state is conserved and then define an effective interaction acting between nucleons in the model space. Hence, the pion is not treated explicitly in conventional nuclear physics. One of the main motivations of the present study is to expand the model space so that the important role of the pion is explicitly accounted for in order to see the effect of the correlation induced by the pion on nuclear structure [3].

With regard to the importance of the pion, we recall findings concerning few-body systems, which can be treated rigorously without the restriction of model space, explicitly using a realistic nucleon-nucleon interaction that has a short-range repulsive core and a tensor force [4,5]. The calculations are performed in a non-relativistic framework, and hence the pion appears mainly as a tensor force, which is much stronger than the central force in the one-pion exchange interaction. There are many existing extensive calculations that employ variational principles and sophisticated numerical techniques, and the calculated results compare with experiments very well [4,5]. The calculated results demonstrate the dominant role of the tensor force in few-body systems. Recent variational calculations carried out by the Argonne group up to $A = 8$ also demonstrate the dominant role of the pion, which amounts to 70–80% of the strength of the two-body interaction part in $3 \leq A \leq 8$ systems.

There are several sets of experimental data that make apparent the important role of the pion in medium and heavy nuclei. The (p, n) reactions with medium and heavy nuclei demonstrate that slightly more than one half of the Gamow-Teller ($\sigma\tau$) strength is carried by 1p-1h excitations, while most of the rest is interpreted as being carried by 2p-2h excitations, due to the coupling with the 1p-1h states by the strong tensor force, which distributes the strength up to at least 50 MeV in excitation energy [6]. A further dramatic result is that the ratio of the longitudinal and transverse spin responses is found to be close to 1, while the strong pionic correlations should provide a large enhancement in the longitudinal channel [7]. This experimental result seems to imply the need for a special

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improvement of the treatment of the pion in nuclear spin response.

The above considerations seem to indicate the importance of the correlation induced by the pion in the description of nuclear many-body systems. For this reason, we would like to construct a framework in which the pionic effect is treated explicitly under a mean-field approximation for medium and heavy nuclei. To this end, we break the parity, spin, and isospin symmetries of single-particle states to treat the pion in the mean-field approximation. This treatment may oppose the common sense belief that pion condensation does not occur in nuclear matter at the saturation density. This fact does not mean, however, that the pion mean field vanishes in a finite nuclear system. The pion is a pseudoscalar meson; it couples with a nucleon through the $\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}$ coupling, and therefore the source term of the pion field needs parity mixing and spin-density modulation. In infinite matter in a high-density region, we have to provide this spin-density modulation (ALS structure) for the pion to act, which requires a large amount of energy [8]. In a finite nuclear system, we have the nuclear surface, and automatically there arises the spin-density modulation necessary for the pion to act. Details are published in ref. [3].

2 Relativistic mean-field theory with a pion

We take the standard expression for the relativistic mean-field Lagrangian together with the pion field. We give here only the equations of motion for the nucleon and the pion explicitly. They are

$$[i\gamma^\mu\partial_\mu - M - g_\pi \boldsymbol{\nabla}\pi\gamma_5\boldsymbol{\gamma}\tau_0]\psi = 0 \quad (1)$$

for the nucleon and

$$(\boldsymbol{\nabla}^2 - m_\pi^2)\pi = -g_\pi\boldsymbol{\nabla}\langle\bar{\psi}\gamma_5\boldsymbol{\gamma}\tau_0\psi\rangle \quad (2)$$

for the pion. Here, the brackets $\langle\cdots\rangle$ denote the ground-state expectation value. The other mesons obey equations of motion similar to those above. Their explicit forms are given in ref. [9].

Equations (1) and (2) describe the structure of the pion mean field. These equations reveal why we have not included the pion mean field until now. If single-particle states in a mean field have good parity, the source term of the pion field becomes zero. Hence, we must break parity in the construction of the single-particle states. The violation of parity is caused by the pion term in the above Dirac equation for nucleons. The pion field is finite when the source term breaks parity symmetry. The pion field is enhanced by the spatial dependence of the source term due to the derivative of the coupling of the pion and the nucleon. When the pion field is finite in (1), the nucleon single-particle state breaks the parity symmetry, which makes the pion source term finite. The self-consistency condition is used to obtain a convergent solution to the above equations. If the pion mean field becomes finite, the parity symmetry is broken, and this is interpreted as the occurrence of pion condensation in the finite nuclei.

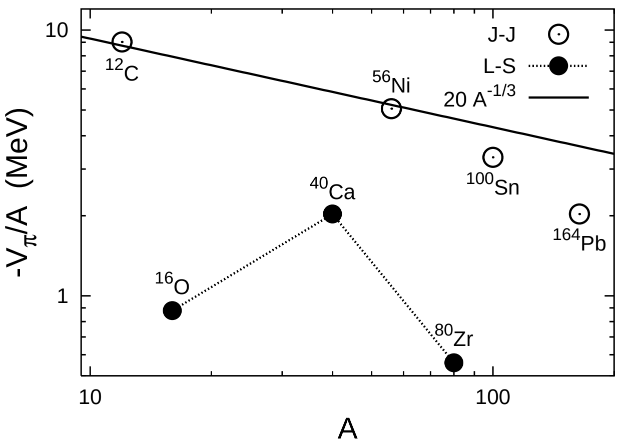

Fig. 1. The pion energy per nucleon as a function of the mass number in the log-log plot. There are two groups; one is for the jj-closed-shell nuclei denoted by open circles and the other is for the LS-closed-shell nuclei denoted by closed circles. The pion energy per nucleon for the jj-closed-shell nuclei decreases monotonically and follows more steeply than $A^{-1/3}$, which is shown by the solid line.

Hence, the single-particle state is expressed as

$$\psi_{njm} = \sum_\kappa \begin{pmatrix} iG_{n\kappa}\mathcal{Y}_{\kappa m} \\ F_{n\kappa}\mathcal{Y}_{\bar{\kappa}m}\end{pmatrix} = \begin{pmatrix} iG_{n\kappa}\mathcal{Y}_{\kappa m} + iG_{n\bar{\kappa}}\mathcal{Y}_{\bar{\kappa}m} \\ F_{n\kappa}\mathcal{Y}_{\bar{\kappa}m} + F_{n\bar{\kappa}}\mathcal{Y}_{\kappa m}\end{pmatrix}. \quad (3)$$

Here, $\mathcal{Y}_{\kappa m}$ is the eigenfunction of the total angular momentum $j = l + s$ and $\mathcal{Y}_{\bar{\kappa}m} = \boldsymbol{\sigma}\cdot\hat{r}\mathcal{Y}_{\kappa m}$. We assume spherical symmetry (jm is a good quantum number) for the intrinsic state. G and F are the radial parts of the single-particle wave function. The summation over κ represents the parity mixing, where κ is $\kappa = -(l_\downarrow + 1)$ for $l_\downarrow = j - 1/2$ and $\kappa = l_\uparrow$ for $l_\uparrow = j + 1/2$.

3 Numerical results with a finite pion field

We present here our numerical results. We use the TM1 parameter set of ref. [9] for all the parameters except for the pion-nucleon coupling. For this we use the value of the Bonn-A potential, which corresponds to setting $g_\pi = f_\pi/m_\pi$, with $f_\pi \sim 1$. Since we are especially interested in the appearance of a finite pion mean field and wish to see its effect under the simplest conditions, we ignore the Coulomb term. We carried out calculations for the $N = Z$ closed-shell nuclei ^{12}C, ^{16}O, ^{40}Ca, ^{56}Ni, ^{80}Zr, ^{100}Sn and ^{164}Pb.

We plot the mass number dependence of the pion energy per nucleon in fig. 1. We mention here that the kinetic energy and the sigma and omega energies per particle are almost independent of the mass number. We see, on the other hand, peculiar behavior in the pion energy. The magnitudes of the pion energy are clearly separated into two groups. One group is large, and their common feature is the jj-closed-shell nuclei, the magic number nuclei due to a larger spin-orbit partner (j-upper) being filled. The other group is small, and they are LS-closed-shell nuclei. The pion energy per nucleon for jj-closed-shell nuclei decreases monotonically with the mass number. The rate of this decrease is more rapid than $A^{-1/3}$. This means that the pion mean-field energy behaves proportionally with the nuclear surface or even stronger than that. For this reason, we use the expression of "surface pion condensation". That separation into two groups for the pion energy: LS-closed and jj-closed-shell cases, is related with the spin-orbit splitting. The spin-orbit partner having larger total spin near the Fermi-surface does not contribute to the total energy for the case of surface pion condensation in the LS-closed-shell nuclei [3].

We discuss here the examples of the qualitative consequence of finite pion mean field. First, we discuss the Gamow-Teller (GT) transitions. Without pion condensation, there exist no transitions for LS-closed-shell nuclei as ^{16}O, ^{40}Ca and only two transitions, for example, for ^{90}Zr. However, the mixing of parity in the intrinsic state allows transitions of 2p-2h states. This makes the spectrum of the GT transitions with some GT strengths above the two dominant peaks in ^{90}Zr [6]. Hence, naturally we have strengths in the region of the higher excitation energy in the mean-field theory with the pion as the experiment demands. Second, the longitudinal spin response functions, which are caused by the pionic correlations, should be largely modified due to pion condensation. Since the large pionic strength is used up to construct the nuclear ground state, the pionic fluctuation ought to be reduced largely. This should make the spin response in the pion channel weak. This remains to be demonstrated in the future work.

Until now phenomena involving high-momentum components have been assigned as nuclear correlations induced by the repulsive core and the tensor force. Surface pion condensation provides us with the possibility to describe the part of the correlation effect induced by the tensor force. This fact implies that we are now able to separate the short-range correlation phenomena into that due to the pion and that due to the repulsive core. In fact, as seen above in the investigation of the parity projection, the surface pion condensation provides a large amount of the 2p-2h excitations in the nuclear ground state automatically. There should be many other consequences of surface pion condensation in nuclear phenomena. The pairing correlations and the spin-orbit couplings are all surface phenomena and surface pion condensation should couple with these correlations and provide rich phenomena.

4 Conclusion

We have studied the possible existence of a finite pion mean field in finite nuclei by introducing a pion field into the relativistic mean-field (RMF) theory. We have extended the RMF theory by introducing the parity-mixed single-particle basis to accommodate the finite pion mean field. We have employed the TM1 parameter set in the RMF theory and introduced the pion field in the pseudovector coupling with the nucleon. With the use of the pion-nucleon coupling constant in free space, we have carried out calculations for $N = Z$ closed-shell nuclei and demonstrated the actual appearance of a finite pion mean field. We have shown that the potential energy associated with the pion behaves roughly proportional to the nuclear surface. For this reason, we refer to the onset of the finite pion mean field as "surface pion condensation". We have given qualitative discussion on the consequences of surface pion condensation on the Gamow-Teller strengths, the spin response functions, and the short-range correlations.

We would like to stress here that we are in the initial stage of our investigation of the role of the pion in finite nuclei. We must carry out various studies in order to establish the mean-field theory with the inclusion of the pion, pursued in this paper for medium and heavy nuclei. We must introduce the rho-meson tensor coupling term, which acts to oppose the appearance of a pion finite mean-field. We should also include the delta-isobar pion coupling terms, which favor a finite pion mean field. In addition, we must study carefully the effect of the short-range repulsions, the so called g' term in the non-relativistic framework. We would like also to work out the exchange terms, *i.e.* the Fock terms in the relativistic many-body theory. We then have to conduct a parameter search for the coupling constants in the Lagrangian. We shall end up with the reduction of the sigma and omega coupling constants, which enhances the effect of the pion mean field. We should further perform a study of the parity projection in the framework of the variation after projection (VAP). There are many studies to be carried out in order to establish the existence of surface pion condensation. We are at the gate of exploring both theoretically and experimentally the phenomena caused by the most important boson, the pion, in nuclear physics.

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